Some Notes on Financial Modeling

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1 Introduction

Applications of methods of statistical physics to financial markets are very popular these days. General idea is that market participant actions determine prices like particle interactions determine thermodynamics. This is rather far fetched however. Though similarity in behaviour of rational and intelligent traders with that of molecules is disputable, the fact that the number of traders say $10^5 - 10^6$ is negligible in comparison with characteristic thermodynamic number which is of order of 10^{23} (Avogadro number) makes this analogy questionable at least.

Actually this means that talking about renormalization group, scaling, phase transitions etc... regarding financial market makes no sense and is complete BS [1]. Nevertheless the application of physical ideas to financial market does make sense but in the context of dynamical systems and multiple time scales.

2 Observables

Every event affecting market microstructure (setting or cancellation of order, transaction etc...) is recorded and stored in proper data base. These events (tick-by-tick data) are actually financial market observables and should be the base of all theories. These data are not regularly spaced in time and their time distribution is very important. Aggregating of data, filtering, subsampling, averaging and all that in attempt to make them regular for

[1] BS may stand for BullShit or Black-Scholes depending on context

the sake of simplicity and applicability standard algorithms leads to loosing information and hence unacceptable.

The main difference between physics and financial market theory is that financial market observables (raw tick-by-tick data) are generally not available though they do exist[2].

Let us suppose that we have access to raw data say for one day, which may be a table with first column containing ticks and second column containing corresponding returns for some asset. We shall see immediately that both ticks and returns vary irregularly on scale of a fraction of second. These are our fast variables. They are actually so fast that may be considered stochastic on a larger time scale (a hour for example). We may suppose that returns are represented by BS process with volatility σ and ticks are distributed somehow (e.g. exponentially) with parameter λ which we call activity.

3 Dynamical model

Reasonable dynamical model which determines dependency of σ and λ on time, might be Lotka-Volterra system.

$$\begin{aligned} \frac{d\sigma}{dt} &= -\sigma(a - b\lambda) \\ \frac{d\lambda}{dt} &= \lambda(c - d\sigma) \end{aligned} \tag{1}$$

Here parameters a, b, c, d considered to be constant on given time scale, but may depend on time on larger time scales (to describe seasonality patterns).

This model is chosen as simplest nonlinear one. Surely more general dynamic models are possible but they need more parameters to fit. Note that this is pure dynamic model, though stochastic models of such type are known in the literature.

[2]Another important difference is: if physicist find something good he publishes it, financial theorist on the contrary publishes only what is not working (saving working things for himself to make money.

4 Variable time step

let P be usual BS propagator

$$P(x,t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{x^2}{2\sigma^2 t}} \tag{2}$$

If we fix some exponential distribution of time steps like

$$f(t) = \lambda e^{-\lambda t} \tag{3}$$

and make an average

$$< P(x) >= \int_0^\infty P(x,t) f(t) dt = \frac{1}{2a} e^{-\frac{|x|}{a}} \tag{4}$$

where $a = \sigma / \sqrt{2\lambda}$. So not uniform time stepping yields heavy tails after averaging.

5 Summary

The following is stated:

1. raw data access is necessary for serious work;
2. time scale hierarchy should be fixed;
3. only fast variables (smallest time scale) are stochastic, all other pure dynamical;
4. averaging over non-uniform time stepping may lead to heavy tails.